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Joint design of stochastically safe setpoints and controllers for non-linear constrained systems by means of optimization

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J. Gillis, G. Horn, M. Diehl

Electrical Engineering (ESAT-STADIUS), KU Leuven, Belgium
Department of Microsystems Engineering (IMTEK), Freiburg Univ, Germany

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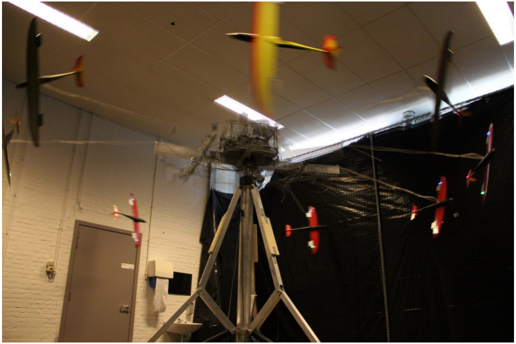

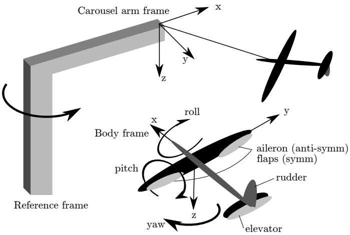

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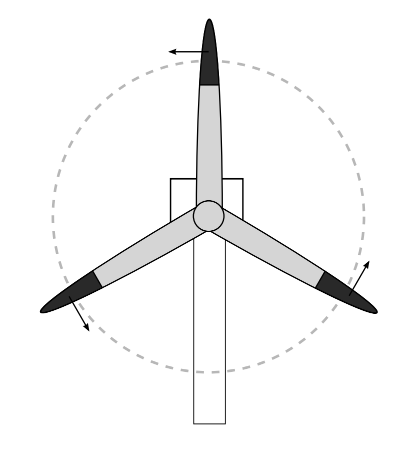

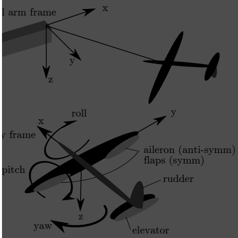

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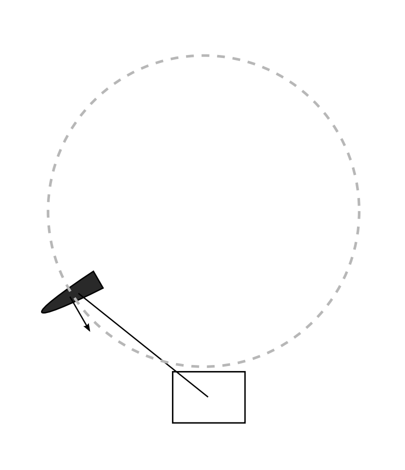

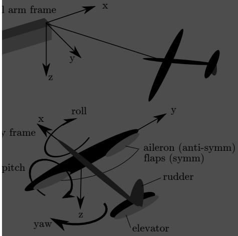

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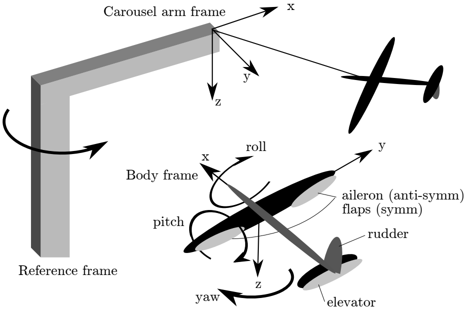

Model

$$\dot{x} = f(x, u, w)$$

- states $x \in \mathbb{R}^{n=18}$
- controls $u \in \mathbb{R}^{m=6}$
- For now: $w \equiv 0$.

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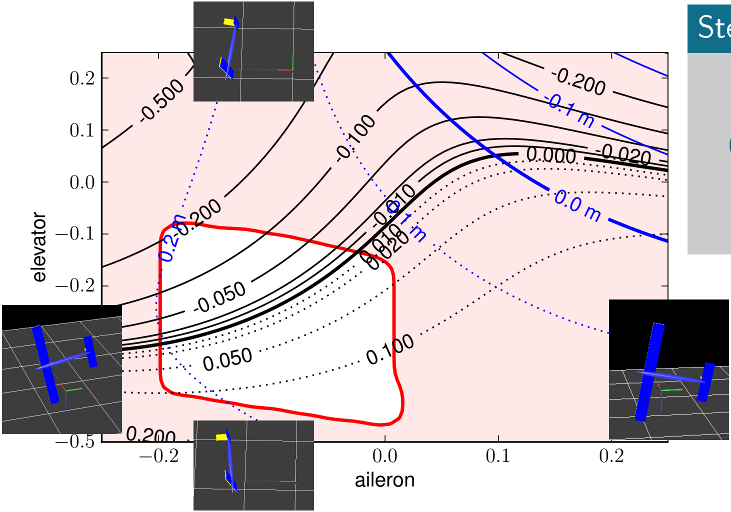

Steady state

$$0 = f(x^\star, u^\star)$$

$$x^\star = S(u^\star)$$

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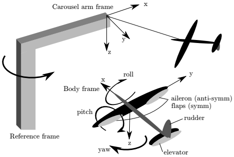
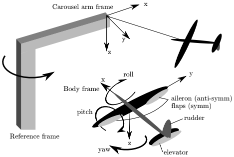


Control bounds

$$u \in \mathbb{U}$$

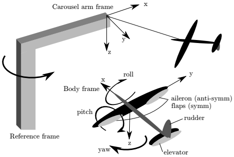
$$\mathbb{U} = \{u | q_j(u) \leq 0\}$$



Operational bounds

$$x \in \mathbb{X}$$

$$\mathbb{X} = \{x | h_i(x) \leq 0\}$$

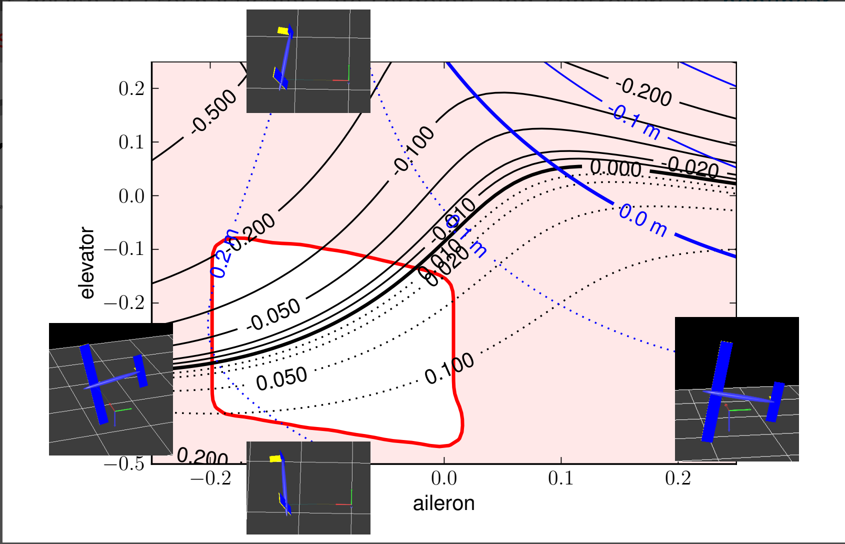


Operational bounds

$$x \in \mathbb{X}$$

Angle of attack [deg]	$[-4.5, 8.5]$
Sideslip angle [deg]	$[-9, 9]$
Airplane position (z) [m]	$]-\infty, 1.5]$
Carousel motor torque [Nm]	$[-20, 20]$
Control surfaces angle [rad]	$[-0.20, 0.20]$
Lift coefficient [-]	$[-0.15, 1.5]$
Winch motor torque [Nm]	$[-78, 78]$
Tether tension [N]	$[0, 600]$
Airspeed. [m/s]	$[10, 65]$

Joint design of stochastically safe setpoints and controllers for nonlinear cons



Joint design of stochastically safe setpoints and controllers for nonlinear constrained systems by means of optimization

Goal

For a range of different tether lengths (r) of the carousel:

- Find *safe* setpoints to fly
- Construct a simple controller to make the setpoints even more *safe*

Safe?

Stay clear from operational bounds *and* stable dynamics

Asymptotically safe setpoint ($x^\star$, $u^\star$)

1. Steady-state: $0 = f(x^\star, u^\star, 0)$
2. Control bounds met: $u^\star \in \mathbb{U}$.
3. $x^\star$ in interior of operational bounds $\mathbb{X}$.
4. Open-loop stable: eigenvalues of $A = \frac{\partial f}{\partial x}(x^\star, u^\star)$ in left half-plane.

Asymptotically safe setpoint $(x^\star,\ u^\star)$

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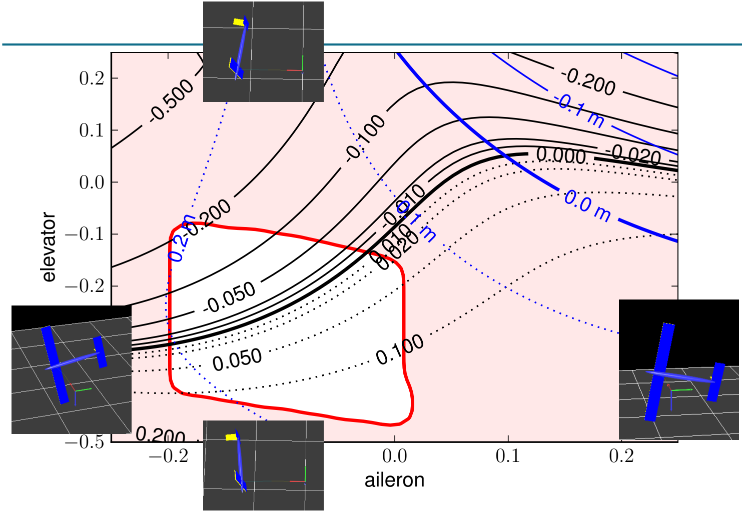
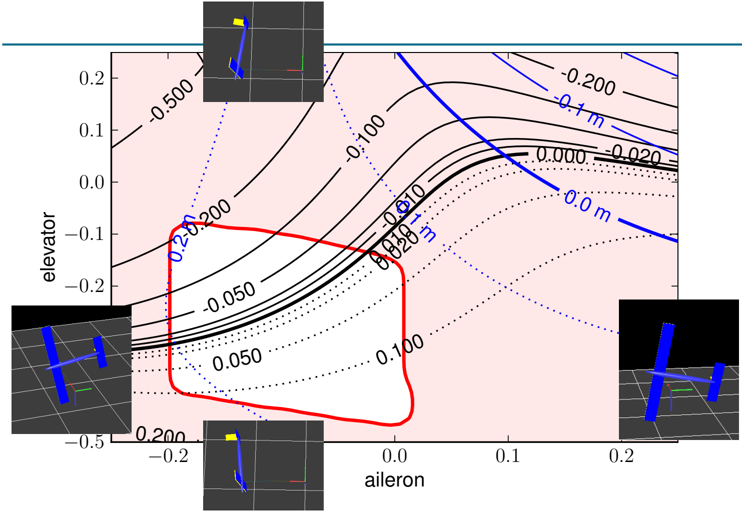
Interpretation

Unleash the airplane in a state close to $x^\star$ and it will not violate any operational bounds while the state evolves to $x^\star$.

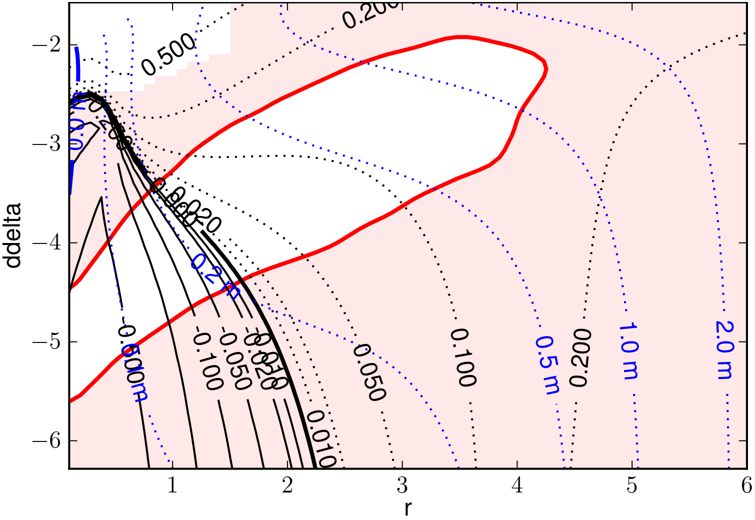

elevator
aileron
-0.500
-0.200
-0.100
-0.050
-0.020
-0.010
0.000
0.010
0.020
0.050
0.100
0.200
-0.1 m
0.0 m
0.1 m
0.2 m
0.2
0.1
0.0
−0.1
−0.2
−0.5
−0.2
0.0
0.1
0.2



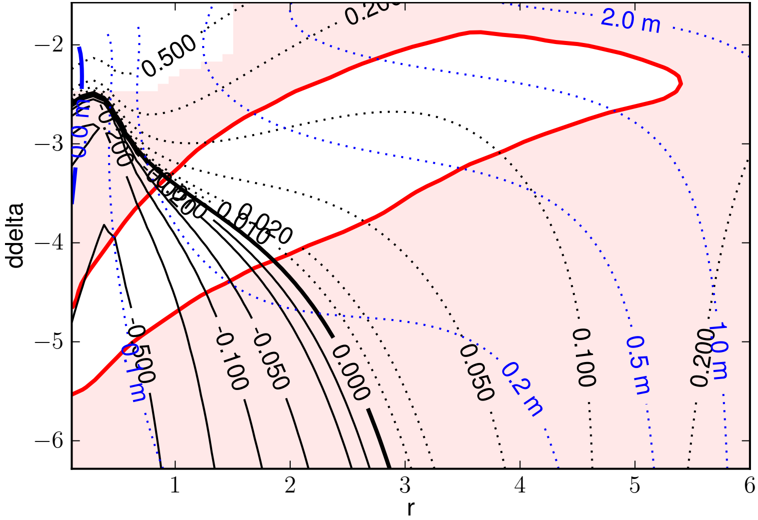
Caveat:

This is only for $r = 1[\mathrm{m}]$

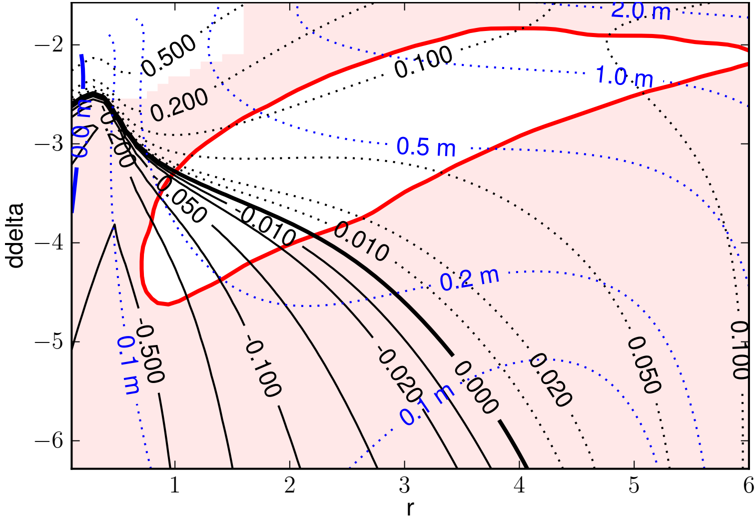
Elevator -0.20



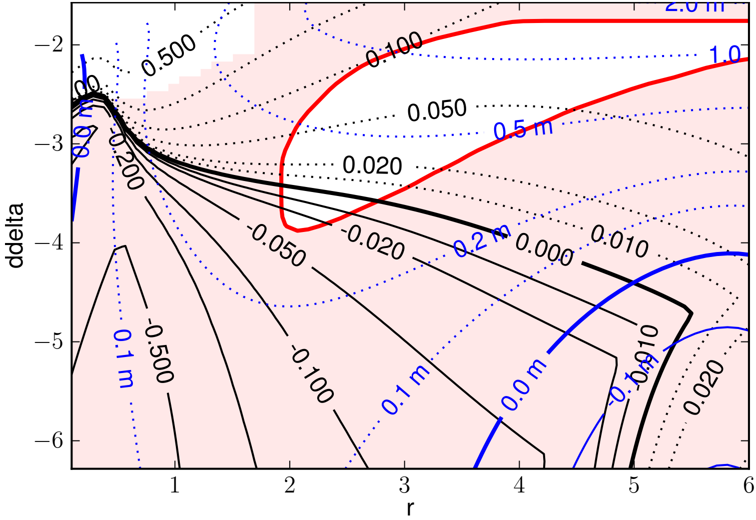
Elevator -0.15



Elevator -0.10



Elevator -0.05

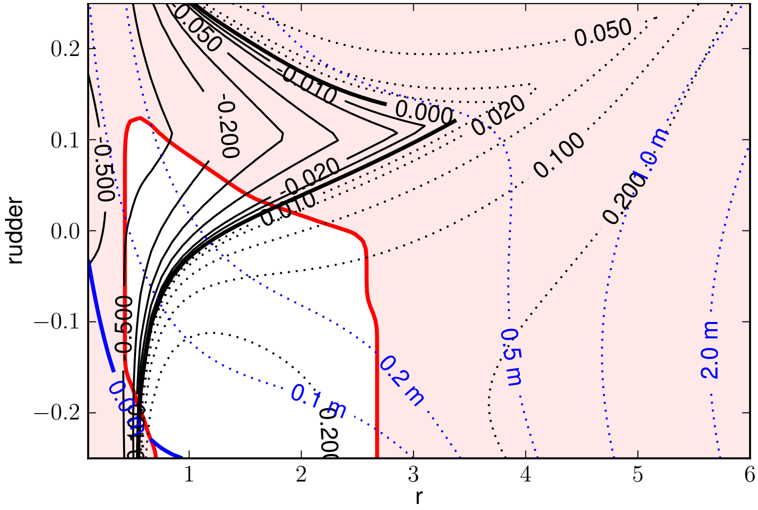


Conclusion from manual inspection

Start with a low value for elevator, and increase it as you roll out the tether.

But there's still other controls ...

e.g. rudder



Problem

- Need some sort of trade-off between stability margin and operational bounds
- Hard to explore the feature-rich set of asymptotically safe setpoints. Can we automate this?

Observation

Due to disturbances, the system never reaches true steady state.

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Disturbance model

$$\dot{x} = f(x, u, w)$$

- White Gaussian disturbances $E[w] = 0$, $\text{cov}[w] = \Sigma_w$
- For $u \equiv u^\star$, we have $E[x] = x^\star$

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Behaviour around steady state $\hat{x} = x - x^\star$

$$\dot{\hat{x}} = \quad A\hat{x} + B\hat{u} + Dw \tag{1}$$

Algebraic Lyapunov Equation

$$\underbrace{AP + PA^T}_{\text{sink}} + \underbrace{D\Sigma_w D^T}_{\text{source}} = 0$$

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State covariance

When A is stable, ALE admits a positive definite solution, and can be interpreted as: $P = \mathrm{cov}[x]$

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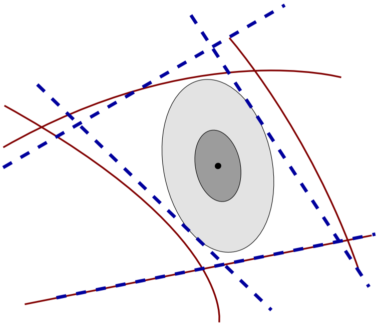
Covariance of bounds

$$\mathrm{cov}[h_i(x)] = \frac{\partial h_i}{\partial x} P \frac{\partial h_i}{\partial x}^T \qquad \text{(scalar)}$$

Towards a robustified operational set

$$h_i(x) + \gamma \sqrt{\text{cov}[h_i(x)]} \leq 0$$



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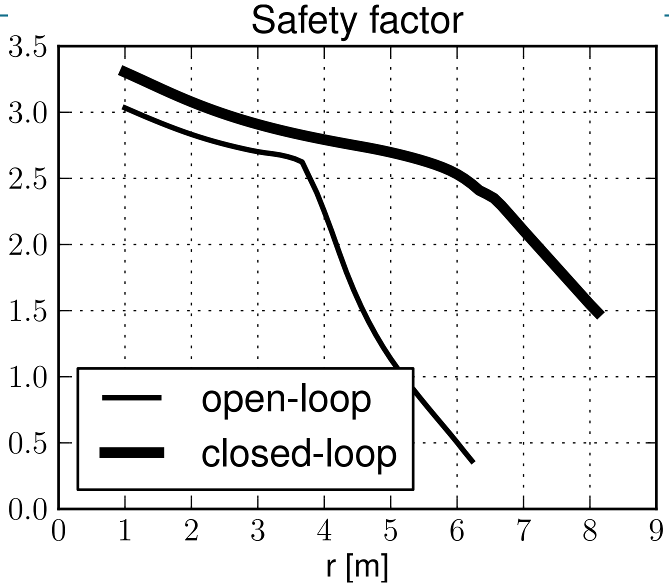
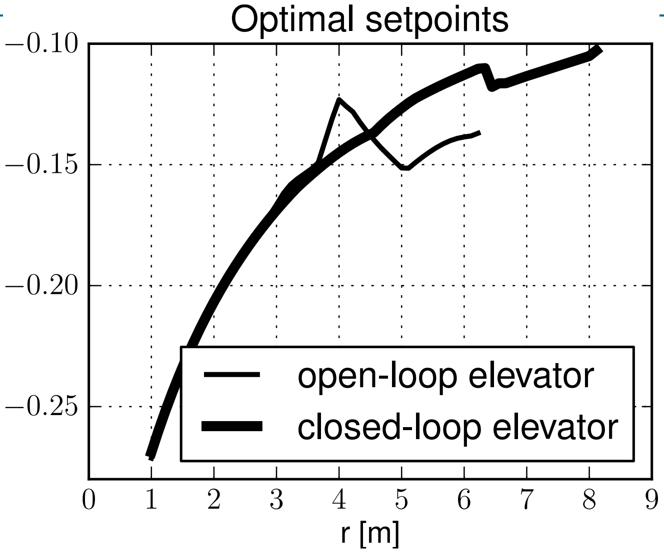
Optimal stochastically safe setpoints

Minimize the chance of violating the operational bounds

$$
\begin{aligned}
\underset{x,u,\gamma,P}{\text{minimize}} \quad & -\gamma \\
\text{subject to} \quad & f(x,u,0) && = \quad 0 \\
& u && \in \quad \mathbb{U} \\
& h_i + \gamma\sqrt{\frac{\partial h_i}{\partial x} P \frac{\partial h_i}{\partial x}^T} && \leq \quad 0 \\
& A(x,u)P + PA^T(x,u) + D(x,u)\Sigma_w D^T(x,u) && = \quad 0.
\end{aligned}
$$


Safety factor
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
6
7
8
9
r [m]
open-loop
closed-loop


Optimal setpoints
−0.10
−0.15
−0.20
−0.25
0
1
2
3
4
5
6
7
8
9
r [m]
open-loop elevator
closed-loop elevator

Add a state-observer

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x} + B\hat{u} + Dw \\ \hat{y} &= Y\hat{x} + v,\end{aligned}$$

Add a state-observer

$$
\begin{aligned}
\dot{\hat{x}} &= \quad A\hat{x} + B\hat{u} + Dw \\
\hat{y} &= \qquad\qquad\quad Y\hat{x} + v,
\end{aligned}
$$

Add a linear feedback-control

$$\hat{u} = K(Y\hat{x} + v)$$

Joint design of stochastically safe setpoints and controllers for nonlinear constrained systems by means of optimization

$$\underset{x,u,\gamma,P,K}{\text{minimize}} \quad -\gamma \tag{2a}$$

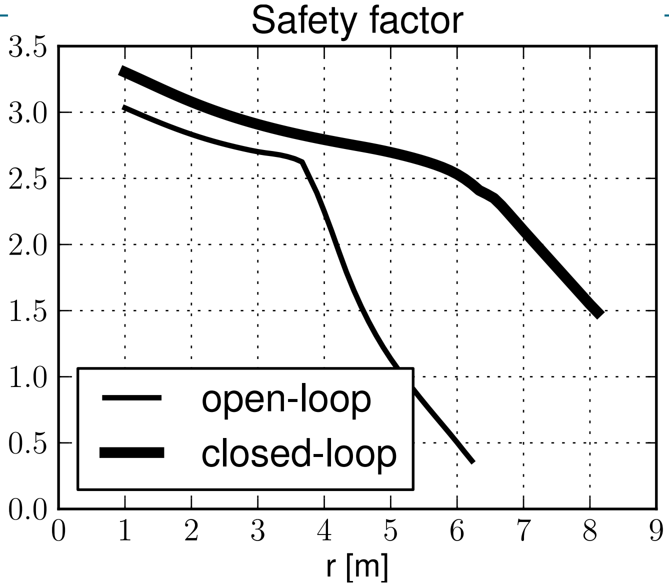
$$\text{subject to} \quad f(x,u,0) = 0 \tag{2b}$$

$$u \in \mathbb{U} \tag{2c}$$

$$h_i + \gamma \sqrt{\frac{\partial h_i}{\partial x} P \frac{\partial h_i}{\partial x}^T} \leq 0 \tag{2d}$$

$$\begin{aligned}(A+BKY)P + P(A+BKY)^T \\ + D\Sigma_w D^T + BK\Sigma_v K^T B^T\end{aligned} = 0 \tag{2e}$$

$$
\begin{aligned}
\underset{x,u,\gamma,P,K}{\text{minimize}} \quad & -\gamma && && \text{(3a)}\\
\text{subject to} \quad & f(x,u,0) && = && 0 \quad \text{(3b)}\\
& q_j + \gamma\sqrt{\frac{\partial q_j}{\partial u} K Y P Y^T K^T \frac{\partial q_j}{\partial u}^T} && \leq && 0 \quad \text{(3c)}\\
& h_i + \gamma\sqrt{\frac{\partial h_i}{\partial x} P \frac{\partial h_i}{\partial x}^T} && \leq && 0 \quad \text{(3d)}\\
& (A+BKY)P + P(A+BKY)^T \\
& + D\Sigma_w D^T + BK\Sigma_v K^T B^T && = && 0 \quad \text{(3e)}
\end{aligned}
$$


Safety factor
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
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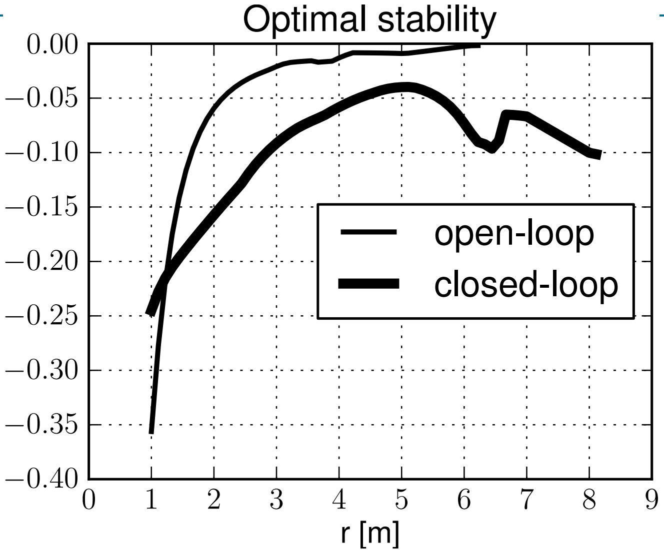
4
5
6
7
8
9
r [m]
open-loop
closed-loop


Optimal stability
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
−0.35
−0.40
0
1
2
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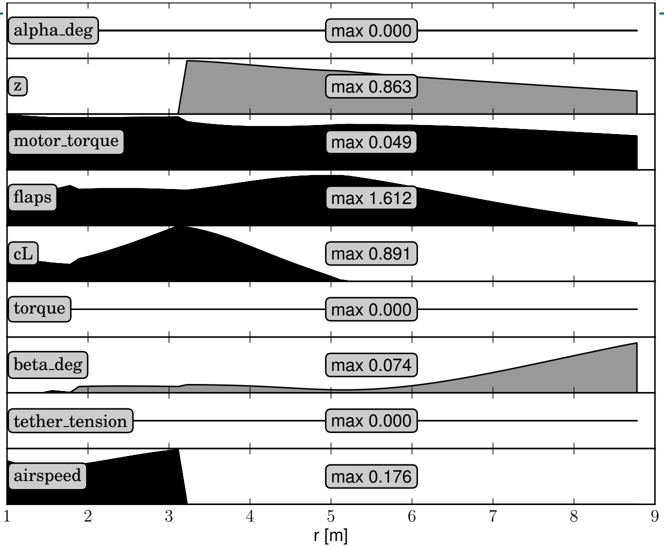
4
5
6
7
8
9
r [m]
open-loop
closed-loop


alpha_deg
max 0.000
z
max 0.863
motor_torque
max 0.049
flaps
max 1.612
cL
max 0.891
torque
max 0.000
beta_deg
max 0.074
tether_tension
max 0.000
airspeed
max 0.176
1
2
3
4
5
6
7
8
9
r [m]

Framework

Python interface of CasADi for symbolics, automatic differentiation

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Solver

IPOPT and WORHP, with linesearch failure if P_{k+1} is not positive definite

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Model is really DAE

Special treatment of invariants in reduced ODE

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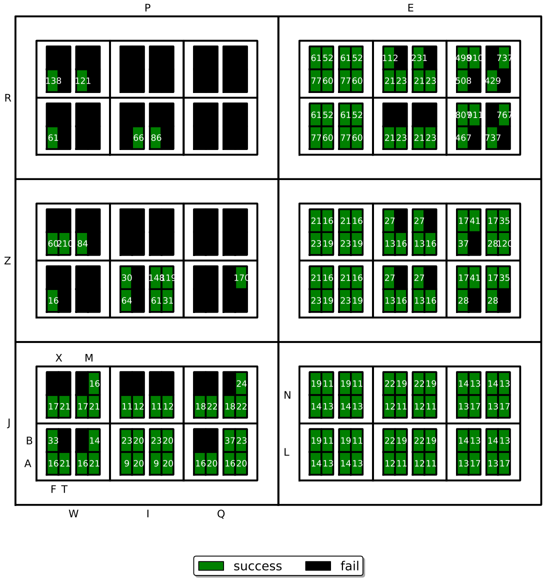
Model is really DAE

Special treatment of invariants in reduced ODE

Elimination of P

Leads to better convergence

Number of iterations (.......)



Summary

- Proposed terminology (asymptotically safe setpoint)
- Formulation to select the safest setpoint
- Formulation to select the safest setpoint + controller
- Solveable by general purpose software (Python/CasADi + IPOPT/WORHP)

Summary

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- Formulation to select the safest setpoint
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Discussion

- Other performance metrics than system integrity possible
- Linearization error should be quantified
- Schur decomposition to solve Lyapunov: $O(n^6) \rightarrow O(n^3)$